Logarithmic conformal field theory approach to topologically massive gravity

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Abstract

We study the topologically massive gravity at the chiral point (chiral gravity) by using the logarithmic conformal field theory. Two new tensor fields of ψ^{new} and X are introduced for a candidate of propagating physical field at the chiral point. However, we show that (ψ^{new}, ψ^L) form a dipole ghost pair of unphysical fields and X is not a primary. This implies that there is no physically propagating degrees of freedom at the chiral point.

1   Introduction

The gravitational Chern-Simons terms in three-dimensional Einstein gravity produces a physically propagating massive graviton [1]. This theory with a negative cosmological constant $\Lambda = -1/l^2$ gives us the AdS$_3$ solution. At the chiral point of $\mu l = 1$, that is, at the chiral gravity, the massive graviton becomes a massless left-moving graviton, an unphysical propagation [2, 3]. However, several counterexamples have been reported [4, 5, 6, 7, 8, 9, 10]. Especially, we wish to mention two cases. Firstly, the non-chiral solution $\psi_{\mu\nu}^{new} = \partial_{\mu l}\psi_{\mu\nu}^{M}|_{\mu l=1}$ was found in global coordinates [5] but it turned out to not satisfy the asymptotic boundary condition for chiral gravity [11]. Secondly, a related family of $X_{\mu\nu} = \bar{L}_{-1}\psi_{\mu\nu}^{new}$ does satisfy the asymptotic boundary condition [6] but it acts as a trivial gauge degree of freedom [11].

In this work, we address this issue again because a key point in the chiral gravity is to find physically propagating degrees of freedom on the AdS$_3$ spacetime background. We show that there is no physically propagating degrees of freedom at the chiral point by using the logarithmic conformal field theory (LCFT). Our result supports that the original work of chiral gravity [2] is correct.

2   Topologically massive gravity

We start with the topologically massive gravity in anti-de Sitter spacetimes (TMG) [1]

$$I_{\mathrm{TMG}} = \frac{1}{16\pi G_3}\int d^3x\sqrt{-g}\left[R + \frac{2}{l^2} - \frac{1}{2\mu}\varepsilon^{\lambda\mu\nu}\Gamma^{\rho}_{\lambda\sigma}\left(\partial_{\mu}\Gamma^{\sigma}_{\nu\rho} + \frac{2}{3}\Gamma^{\sigma}_{\mu\tau}\Gamma^{\tau}_{\nu\rho}\right)\right], \tag{1}$$

where ε is the tensor density defined by $\epsilon/\sqrt{-g}$ with $\epsilon^{012} = 1$. The $1/\mu$-term is the first higher derivative correction in three dimensions because it is the third-order derivative.

Varying the this action leads to the Einstein equation

$$G_{\mu\nu} + \frac{1}{\mu}C_{\mu\nu} = 0, \tag{2}$$

where the Einstein tensor including the cosmological constant is given by

$$G_{\mu\nu} = R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} - \frac{1}{l^2}g_{\mu\nu} \tag{3}$$

and the Cotton tensor is

$$C_{\mu\nu} = \varepsilon_{\mu}^{\ \alpha\beta}\nabla_{\alpha}\left(R_{\beta\nu} - \frac{1}{4}g_{\beta\nu}R\right). \tag{4}$$

The BTZ black hole solution [12] is given by

$$ds_{BTZ}^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\phi + N^{\phi}(r)dt\right)^2, \tag{5}$$

2

where the metric function f and the lapse function N^ϕ are

$$f(r) = -8G_3 m + \frac{r^2}{l^2} + \frac{16G_3^2 j^2}{r^2}, \quad N^\phi(r) = -\frac{4G_3 J}{r^2}. \tag{6}$$

Here m and j are the mass and angular momentum of the BTZ black hole, respectively. In this work, we consider the AdS$_3$ solution which appears for $m = -1/8G_3$, $j = 0$ only. We note that the Cotton tensor $C_{\mu\nu}$ vanishes for any solution to Einstein gravity, so all solutions to general relativity are also solutions of TMG.

However, TMG possesses physical degrees of freedom propagating on the AdS$_3$ spacetimes. In order to explore this, we study the perturbation around the AdS$_3$ spacetimes by considering the metric fluctuations

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \tag{7}$$

where the AdS$_3$ metric appears as

$$ds^2_{AdS} = \bar{g}_{\mu\nu}dx^\mu dx^\nu = -\left(1 + \frac{r^2}{l^2}\right)dt^2 + \frac{dr^2}{\left(1 + \frac{r^2}{l^2}\right)} + r^2 d\phi^2. \tag{8}$$

Introducing the global coordinates with $r = l\sinh\rho$ and $\tau = lt$ as

$$ds^2_{global} = \bar{g}_{\mu\nu}dx^\mu dx^\nu = l^2\left(-\cosh^2\rho d\tau^2 + \sinh^2\rho d\phi^2 + d\rho^2\right), \tag{9}$$

then it covers the whole space with the boundary at $\rho = \infty$. The isometry group of AdS$_3$ space is $SL(2,R) \times \overline{SL(2,R)}$ and its generators are realized on scalar fields as

$$L_0 = i\partial_u, \quad L_{\pm 1} = ie^{\pm iu}\left(\frac{\cosh 2\rho}{\sinh 2\rho}\partial_u - \frac{1}{\sinh 2\rho}\partial_v \mp \frac{i}{2}\partial_\rho\right), \tag{10}$$

$$\bar{L}_0 = i\partial_v, \quad \bar{L}_{\pm 1} = ie^{\pm iv}\left(\frac{\cosh 2\rho}{\sinh 2\rho}\partial_v - \frac{1}{\sinh 2\rho}\partial_u \mp \frac{i}{2}\partial_\rho\right), \tag{11}$$

where two light-cone coordinates $u/v = \tau \pm \phi$ are introduced to study the boundary physics. Using the transverse and traceless gauge $\bar{\nabla}_\alpha h^{\alpha\beta} = 0$ with $h = 0$, the linearized equation of $\mathcal{G}^{(1)}_{\mu\nu} + \mathcal{C}^{(1)}_{\mu\nu} = 0$ to the Einstein equation (2) could be converted into a compact form as the graviton equations of motion [2],

$$\left(\mathcal{D}^R \mathcal{D}^L \mathcal{D}^M h\right)_{\mu\nu} = 0 \tag{12}$$

where

$$\left(\mathcal{D}^{L/R}\right)_\mu{}^\nu = \delta_\mu{}^\nu \pm l\varepsilon_\mu{}^{\alpha\nu}\bar{\nabla}_\alpha, \quad \left(\mathcal{D}^M\right)_\mu{}^\nu = \delta_\mu{}^\nu + \frac{1}{\mu}\varepsilon_\mu{}^{\alpha\nu}\bar{\nabla}_\alpha. \tag{13}$$

3

Here the covariant derivative $\bar{\nabla}_\alpha$ is defined with respect to $\bar{g}_{\mu\nu}$ in Eq.(9). We note that $\mathcal{D}^R, \mathcal{D}^L, \mathcal{D}^M$ commute with each other. Then we could obtain all solutions of $h_{\mu\nu}^{R/L}$ and $h_{\mu\nu}^M$ which satisfy the first-order equations, respectively,

$$\left(\mathcal{D}^R h^R\right)_{\mu\nu} = 0, \ \left(\mathcal{D}^L h^L\right)_{\mu\nu} = 0, \ \left(\mathcal{D}^M h^M\right)_{\mu\nu} = 0, \tag{14}$$

where the first two equations describe gauge degrees of freedom (right-and left-movers), while the last one describes physically propagating degrees of freedom (massive graviton). However, at the chiral point of $\mu l = 1$, we find from $\mathcal{D}^M = \mathcal{D}^L$ that the massive graviton turns out to be the left-mover. The linearized equation of $\mathcal{G}_{\mu\nu}^{(1)} + \mathcal{C}_{\mu\nu}^{(1)} = 0$ can be rewritten as

$$\left(\mathcal{D}^M \mathcal{G}^{(1)}\right)_{\mu\nu} = 0 \tag{15}$$

with

$$\mathcal{G}_{\mu\nu}^{(1)} = -\frac{1}{2}\left(\bar{\nabla}^2 + \frac{2}{l^2}\right)h_{\mu\nu}. \tag{16}$$

Further, considering

$$\left(\bar{\mathcal{D}}^M \mathcal{D}^M \mathcal{G}^{(1)}\right)_{\mu\nu} = 0 \tag{17}$$

with

$$\bar{\mathcal{D}}^M = \delta_\mu{}^\nu - \frac{1}{\mu}\varepsilon_\mu{}^{\alpha\nu}\bar{\nabla}_\alpha \tag{18}$$

leads to the fourth-order equation

$$\left(\bar{\nabla}^2 + \frac{3}{l^2} - \mu^2\right)\left(\bar{\nabla}^2 + \frac{2}{l^2}\right)h_{\mu\nu} = 0. \tag{19}$$

Eq.(15) could be derived from the bilinear action up to total derivatives as

$$
\begin{aligned}
I_{(2)} &= -\frac{1}{32\pi G_3}\int d^3x\sqrt{-g}h^{\mu\nu}\left(\mathcal{G}_{\mu\nu}^{(1)} + \mathcal{C}_{\mu\nu}^{(1)}\right) \\
&= \frac{1}{64\pi G_3}\int d^3x\sqrt{-g}\left[-\bar{\nabla}^\lambda h^{\mu\nu}\bar{\nabla}_\lambda h_{\mu\nu} - \frac{1}{\mu}\bar{\nabla}_\alpha h^{\mu\nu}\epsilon_\mu{}^{\alpha\beta}\left(\bar{\nabla}^2 + \frac{2}{l^2}\right)h_{\beta\nu} + \frac{2}{l^2}h^{\mu\nu}h_{\mu\nu}\right] (20)
\end{aligned}
$$

On the other hand, we can use the first-order equations in Eq.(14) to have

$$\left(\mathcal{D}^L \mathcal{D}^R h^R\right)_{\mu\nu} = 0, \ \left(\mathcal{D}^R \mathcal{D}^L h^L\right)_{\mu\nu} = 0, \ \left(\bar{\mathcal{D}}^M \mathcal{D}^M h^M\right)_{\mu\nu} = 0, \tag{21}$$

which leads to the conventional second-order equations

$$\left(\bar{\nabla}^2 + \frac{2}{l^2}\right)h_{\mu\nu}^{R/L} = 0, \ \left(\bar{\nabla}^2 + \frac{2}{l^2} + \frac{1}{l^2} - \mu^2\right)h_{\mu\nu}^M = 0. \tag{22}$$

Here the first equation represents the massless gravitons of right and left-movers, while the last one denotes the massive graviton for $\mu^2 \neq 1/l^2$. We note that $\frac{2}{l^2} = -2\Lambda$ is not the mass

4

term but it appears because all gravitons are propagating on the AdS$_3$ spacetimes. For the scalar propagation, there is no such a term. Also, these second-order equations are insensitive to signs of $\pm l$ and $\pm 1/\mu$, while the first-order equations are sensitive to signs [13].

The explicit form of the wave function for a massive primary graviton could be constructed by using $SL(2,R)$-algebra. Considering $\bar{\nabla}^2 = -(2/l^2)(L^2 + \bar{L}^2 + 3)$ acting on a rank two tensor, Eq.(22) leads to two algebraic equations of weights $(h, \bar{h})$ for primary fields

$$h(h-1) + \bar{h}(\bar{h}-1) - 2 = 0, \quad 2h(h-1) + 2\bar{h}(\bar{h}-1) - 3 = \mu^2 l^2. \tag{23}$$

Solving these equations plus the gauge condition together with asymptotic boundary condition, we have primary weights (2,0), (0,2), and $(3/2 + \mu l/2, -1/2 + \mu l/2)$ with $h - \bar{h} = \pm 2$ for left, right-moving massless gravitons, and massive graviton, respectively. The massive primary states are constructed from $L_1 h^M_{\mu\nu} = \bar{L}_1 h^M_{\mu\nu} = 0$ as

$$h^M_{\mu\nu} = \Re \psi^M_{\mu\nu}, \tag{24}$$

where

$$\psi^M_{\mu\nu} = e^{-(3/2+\mu l/2)iu - (-1/2+\mu l/2)iv} \frac{\sinh^2 \rho}{(\cosh \rho)^{1+\mu l}} \begin{pmatrix} 1 & 1 & ia \\ 1 & 1 & ia \\ ia & ia & -a^2 \end{pmatrix}_{\mu\nu} \tag{25}$$

with $a = 2/\sinh 2\rho$. Here we note that the left-moving primary $h^L_{\mu\nu}$ is found when $\mu l = 1$.

3 TMG at the chiral point

We start by noting that at the chiral point, one has $\psi^M_{\mu\nu} = \psi^L_{\mu\nu}$ coupled to (2,0)-operator with spin $s = 2$. In this section, we study a new mode from the massive graviton defined by [5]

$$\psi^{new}_{\mu\nu} = \partial_{\mu l} \psi^M_{\mu\nu}|_{\mu l=1} = y(\tau, \rho)\psi^L_{\mu\nu} \tag{26}$$

with

$$y(\tau, \rho) = -i\tau - \ln[\cosh \rho]. \tag{27}$$

We note that the partial derivative of $\partial_{\mu l}$ is the logarithmic operator to generate the quasi-primary field from the primary field $\psi^M_{\mu\nu}$ [14] and it commutes other operators. Using $L_0 y = \bar{L}_0 y = 1/2$ and $L_1 y = \bar{L}_1 y = 0$, it is easily shown that for a non-chiral field $\psi^{new}_{\mu\nu}$, one has

$$L_0 \psi^{new}_{\mu\nu} = 2\psi^{new}_{\mu\nu} + \frac{1}{2}\psi^L_{\mu\nu}, \quad \bar{L}_0 \psi^{new}_{\mu\nu} = \frac{1}{2}\psi^L_{\mu\nu}, \quad L_1 \psi^{new}_{\mu\nu} = \bar{L}_1 \psi^{new}_{\mu\nu} = 0. \tag{28}$$

while for a chiral field $\psi^L_{\mu\nu}$, these are

$$L_0 \psi^L_{\mu\nu} = 2\psi^L_{\mu\nu}, \quad \bar{L}_0 \psi^L_{\mu\nu} = 0, \quad L_1 \psi^L_{\mu\nu} = \bar{L}_1 \psi^L_{\mu\nu} = 0. \tag{29}$$

This shows that $\psi_{\mu\nu}^{new}$ is not an eigenstate of L_0 and $\bar{L}_0$ and thus it is impossible to decompose it as a linear combination of eigenstates to L_0 and $\bar{L}_0$. Actually $\psi_{\mu\nu}^{new}$ is a logarithmic partner of $\psi_{\mu\nu}^L$. We obtain the equations for ψ^{new}

$$\left(\mathcal{D}^L h^{new}\right)_{\mu\nu} \neq 0, \quad \left(\mathcal{D}^L \mathcal{D}^L h^{new}\right)_{\mu\nu} = 0, \quad \left(\mathcal{D}^R \mathcal{D}^L \mathcal{D}^L h^{new}\right)_{\mu\nu} = 0. \tag{30}$$

Even though ψ^{new} solves the classical equation, it does not provide the conventional equation. We derive the equation of motion from $(\mathcal{D}^R \mathcal{D}^L h^{new})_{\mu\nu} = -\psi_{\mu\nu}^L/l^2$ as a coupled equation

$$\left(\nabla^2 + \frac{2}{l^2}\right)\psi_{\mu\nu}^{new} - \frac{2}{l^2}\psi_{\mu\nu}^L = 0. \tag{31}$$

Combining Eq.(31) with Eq.(22) leads to the fourth-order equation for $\psi_{\mu\nu}^{new}$

$$\left(\nabla^2 + \frac{2}{l^2}\right)^2 \psi_{\mu\nu}^{new} = 0, \tag{32}$$

which is basically different from the second-order equations for $\psi_{\mu\nu}^L$. This fourth-order equation may induce the unitarity problem.

Unfortunately, this field and its descendants diverge linearly in $y(\tau, \rho)$ near the boundary at $\rho = \infty$ [11]. Hence it is problematic to argue that ψ^{new} is a physical mode but not pure gauge at the chiral point.

On the other hand, it was suggested that a field $X_{\mu\nu} = (\bar{L}_{-1}\psi^{new})_{\mu\nu}$ coupled to (2,1)-operator may be a primary field with the correct asymptotic behavior at the chiral point [6], even though $X_{\mu\nu}$ failed to be a truly primary field because of $L_1 X_{\mu\nu} = 0$ and $\bar{L}_1 X_{\mu\nu} = \psi_{\mu\nu}^L$. It was argued that if one defines physical states modulo locally pure-gauge states, $X_{\mu\nu}$ would be a primary field.

However, in the next section, we prove that $\psi_{\mu\nu}^{new}$ is not a physical field and $X_{\mu\nu}$ could not be a primary field by using the logarithmic conformal field theory (LCFT).

4 LCFT and Singleton coupled operator with $s = 0$

It was shown that $\psi_{\mu\nu}^{new}$ is a logarithmic partner of $\psi_{\mu\nu}^L$, implying the LCFT [5, 15]. A LCFT differs from an ordinary CFT in that the Virasoro generator L_0 is not diagonalizable [16]. In addition to the primary and descendant fields, it includes pairs of operators which form the Jordan cell structure for L_0 with highest weights $(h, \bar{h})$

$$L_0|C\rangle = h|C\rangle, \quad L_0|D\rangle = h|C\rangle|D\rangle + |C\rangle, \quad L_n|C\rangle = L_n|D\rangle = 0, \quad n \geq 1$$
$$\bar{L}_0|C\rangle = \bar{h}|C\rangle, \quad \bar{L}_0|D\rangle = \bar{h}|C\rangle|D\rangle + |C\rangle, \quad \bar{L}_n|C\rangle = \bar{L}_n|D\rangle = 0, \quad n \geq 1 \tag{33}$$

which means that the primary field C and the quasi-primary field D are in reducible, but indecomposable representation of the Virasoro algebra. Kogan proposed that a dipole ghost pair (A, B) can represent a singleton, which induces the 2-point function for a pair of logarithmic operators (D, C) with $h = \bar{h}$ [17]. Here C is an operator with (1,1). Also he has shown that this is the origin of logarithmic singularities in the 4-point functions. This logarithmic pair has the 2-point functions [18]

$$
\begin{aligned}
\langle C(\mathbf{x})C(\mathbf{y})\rangle &= 0, \\
\langle C(\mathbf{x})D(\mathbf{y})\rangle &= \frac{c}{|\mathbf{x} - \mathbf{y}|^{2\Delta}}, \\
\langle D(\mathbf{x})D(\mathbf{y})\rangle &= \frac{c}{|\mathbf{x} - \mathbf{y}|^{2\Delta}} \left[d - 2c\ln|\mathbf{x} - \mathbf{y}|\right].
\end{aligned} \tag{34}
$$

Here $\Delta = h + \bar{h}$ is a degenerate dimension of C and D. The coefficient c is determined by the normalization of C and D. However, d is arbitrary and thus it can be set to any value, using the symmetry of the theory under $D \to D + \lambda C$ which leaves Eq.(33) unchanged.

In the $\text{AdS}_3/\text{CFT}_2$ correspondence, there exists a puzzle of missing states between CFT_2 and supergravity [19]. The gauge bosons appear in the resolution of this puzzle. These are chiral primaries [20]. But on the supergravity side, these are absent and thus may be considered as unphysical singletons on AdS_3 [21]. The authors in [22] found that these gauge bosons coupled to (2,0) and (0,2) operators on the boundary receive logarithmic corrections from an AdS_3 scattering calculation. The clearest evidence for the existence of logarithmic operators in $\text{AdS}_3/\text{CFT}_2$ correspondence comes from calculations of the greybody factors in AdS_3 spacetimes. Since the greybody factors are related to the 2-point functions in CFT, logarithms found here are a clear indication that we have logarithmic operators on the boundary. In this sense, it was important to test the relationship between gauge boson and singleton by calculating greybody factors [23].

Correlation functions of operators $\mathcal{O}_i(\mathbf{x})$ in the CFT_2 on the boundary of AdS_3 spacetimes which corresponds to fields Φ_i in the bulk could be calculated from the bulk action I using the relation

$$
\langle e^{\Sigma_i \int d^2 x \Phi_{b,i}(\mathbf{x}) \mathcal{O}_i(\mathbf{x})} \rangle_{CFT} = e^{-I[\{\Phi_i\}]}|_{\Phi_i = \Phi_{b,i}}. \tag{35}
$$

We are now in a position to introduce the action for a dipole ghost pair (A, B) on the AdS_3 spacetimes [18, 17]

$$
I_{\text{DG}} = \frac{1}{16\pi G_3} \int d^3 x \sqrt{-g} \left[\partial_\mu A \partial^\mu B - m^2 AB - \frac{1}{2}B^2\right] \tag{36}
$$

with mass m. It is not clear if this action comes from supergravity theories. Rather, this takes a similar form of the Nakanishi-Lautrup formalism in the gauge theory [25]. In detail,

Eq.(36) with $m^2 = 0$ and $A_\mu = \partial_\mu A$ leads to a gauge-fixing term as

$$I_{\mathrm{GF}} = -\int d^3x \sqrt{g} \left[B\partial_\mu A^\mu + \frac{\alpha}{2} B^2 \right] \tag{37}$$

with $\alpha = 1$. Here B is the Nakanishi-Lautrup field, while A corresponds to σ-field which leads to the negative-norm state. Thus A and B form the zero-norm states.

Their equations of motion are given by

$$\left(\nabla^2 + m^2 \right) A + B = 0, \quad \left(\nabla^2 + m^2 \right) B = 0 \tag{38}$$

which leads to the fourth-order equation for A

$$\left(\nabla^2 + m^2 \right)^2 A = 0. \tag{39}$$

This is the same equation (32) for $\psi_{\mu\nu}^{new}$ [5] upon choosing $\psi_{\mu\nu}^L \sim -B$. A pair of dipole ghost fields (A, B) coupled to (D, C)-operators on the boundary could represent a pair of $(\psi_{\mu\nu}^{new}, \psi_{\mu\nu}^L)$. Hence we may have a correspondence

$$B \leftrightarrow \psi^L, \quad A \leftrightarrow \psi^{new}. \tag{40}$$

Here we observe a slight difference that a scalar B is coupled to a primary operator C with $(1,1)$, whereas a tensor field $\psi_{\mu\nu}^L$ is coupled to a primary operator with $(2,0)$. On the other hand, a scalar A is coupled to a quasi-primary operator D, while a tensor $\psi_{\mu\nu}^{new}$ is coupled to a quasi-primary operator.

In addition, we may derive equations for A and B using the operator method. For a primary field C, from Eq.(33), we have

$$C \sim \frac{e^{-ihu-i\bar{h}v}}{(\cosh \rho)^{h+\bar{h}}} \tag{41}$$

From the condition for D in Eq.(33), we have

$$D = [u + v + f(\rho)]C, \tag{42}$$

where $f(\rho)$ is determined by considering the singleton condition of $L_1|D\rangle = \bar{L}_1|D\rangle = 0$ as

$$f(\rho) = -2i \ln[\cosh \rho] + \delta. \tag{43}$$

Here δ is an arbitrary constant, which can set to any value using the freedom to shift D by an amount proportional to $C(D \to D + \lambda C)$. Finally, we obtain

$$D = [-i\tau - \ln[\cosh \rho] + \delta'](-2iC), \tag{44}$$

8

which is the same relation as in Eq.(26). Evaluating the second Casimirs gives the equations of motion for the bulk fields $\tilde{D}$ and $\tilde{C}$ in AdS$_3$ spacetimes which are related to D and C as

$$\left(\bar{\nabla}^2 + m^2\right)\tilde{D} - 4(\Delta - 1)\tilde{C} = 0, \ \left(\bar{\nabla}^2 + m^2\right)\tilde{C} = 0 \tag{45}$$

with the mass $m^2 l^2 = 2h(h - 1) + 2\bar{h}(\bar{h} - 1) = \Delta(\Delta - 2)$. Comparing the above with Eq.(38), one finds a relation of $\tilde{D} \sim A$ and $\tilde{C} \sim -B$. Here we have $m^2 = 0$ because of $h = \bar{h} = 1(\Delta = 2)$. This is clear since the massless dipole pair of (A, B) corresponds to the massless graviton pair of $(\psi_{\mu\nu}^{new}, \psi_{\mu\nu}^L)$.

We are now in a position to show that $X_{\mu\nu} = \bar{L}_{-1}\psi_{\mu\nu}^{new}$ [6] could not be a primary field at the chiral point. Let us assume that there exist two primary fields C and C'. In the earlier situation in Eq.(33) where two C and D became degenerate, while in this case we have two fields whose dimension differs by an integer N as [24]

$$L_n|C\rangle = 0, n \geq 1 \text{ and } (L_1)^N|D\rangle = \beta|C'\rangle, \ L_n|D\rangle = 0, n \geq 2, \tag{46}$$

where C' is another primary field with conformal weights $(h - N, \bar{h})$ and β is a non-zero constant. In this case, C with $(h, \bar{h})$ is not really a primary field but rather a descendent of C': $|C\rangle = \sigma_{-N}|C'\rangle$, where σ_{-N} is some combination of the Virasoro generators of dimension N with $[L_n, \sigma_{-N}] = 0$, $n \geq 1$. This implies why the 2-point function is still zero. Explicitly, a logarithmic pair C and D still have the same 2-point functions as in Eq.(34). However, C' is just an ordinary primary field with the usual 2-point function

$$\langle C'(u, v)C'(0, 0)\rangle \propto \frac{1}{u^{2(h-N)}v^{2\bar{h}}}. \tag{47}$$

If $\beta \neq 0$, we have (1,1) for C and then (0,1) for C' when $N = 1$. However, this does not belong to $X_{\mu\nu}$ coupled to (2,1)-operator. Furthermore, we could not develop another primary field C' because the condition of $L_1\psi_{\mu\nu}^{new} = 0$ for $N = 1$ implies $\beta = 0$. There is no room to accommodate another primary field at the chiral point except a pair of (ψ^L, ψ^R).

5   Discussions

We investigate whether or not the new tensor fields of $\psi_{\mu\nu}^{new}$ and $X_{\mu\nu}$ become propagating physical fields at the chiral point. This work is important because an urgent work in the chiral gravity is to find physically propagating degrees of freedom on the AdS$_3$ spacetime background using the AdS$_3$/CFT$_2$ correspondence. We show that there is no physically propagating degrees of freedom at the chiral point by using the logarithmic conformal field theory. Explicitly, it is found that (ψ^{new}, ψ^L) form a dipole ghost pair (unphysical fields) as

9

well as X is not a primary field. Hence our result supports that the original work of chiral gravity [2] is correct.

On the other hand, we discuss the unitarity problem related to the logarithmic operators. This is clearly understood from the bulk side. In general, the fourth-order equations for ψ^{new} and A may induce the unitarity problem in calculating their on-shell amplitudes. It is noted that these logarithmic terms originate from the unphysical dipole ghost fields (A, B). As was shown in [25], this pair (A, B) is turned into the zero-norm state by the Goldstone dipole mechanism in Minkowski spacetime. We suggest that the boundary logarithmic terms is related to the negative-norm state of A. In order to remove the negative-norm state, we impose the subsidiary condition as $B^+(x)|0\rangle_{\rm phys} = 0$. Then the physical space($|0\rangle_{\rm phys}$) will not include any A-particle state. This corresponds to the dipole mechanism to cancel the negative-norm state. Similarly, we expect that this prescription may work for the CFT$_2$ on the boundary at infinity.

Fortunately, since the new field $\psi^{new}_{\mu\nu}$ is not a physical field, we do not consider this unitarity problem seriously. As a byproduct of introducing $\psi^{new}_{\mu\nu}$, we show that the chiral gravity is unitary, leaving the right-moving graviton because a pair of $(\psi^{new}_{\mu\nu}, \psi^L_{\mu\nu})$ forms zero-norm state as a dipole ghost pair.

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